NAG Toolbox for MATLAB

s17ah

1 Purpose

s17ah returns a value of the Airy function, $\mathrm{Bi}(x)$, via the function name.

2 Syntax

```
[result, ifail] = s17ah(x)
```

3 Description

s17ah evaluates an approximation to the Airy function $\mathrm{Bi}(x)$. It is based on a number of Chebyshev expansions.

For $x < -5$,

$$\mathrm{Bi}(x) = \frac{a(t)\cos z + b(t)\sin z}{(-x)^{1/4}},$$

where $z = \frac{\pi}{4} + \frac{2}{3}\sqrt{-x^3}$ and $a(t)$ and $b(t)$ are expansions in the variable $t = -2\left(\frac{5}{x}\right)^3 - 1$.

For $-5 \leq x \leq 0$,

$$\mathrm{Bi}(x) = \sqrt{3}(f(t) + xg(t)),$$

where f and g are expansions in $t = -2\left(\frac{x}{5}\right)^3 - 1$.

For $0 < x < 4.5$,

$$\mathrm{Bi}(x) = e^{11x/8}y(t),$$

where y is an expansion in $t = 4x/9 - 1$.

For $4.5 \leq x \leq 9$,

$$\mathrm{Bi}(x) = e^{5x/2}v(t),$$

where v is an expansion in $t = 4x/9 - 3$.

For $x \geq 9$,

$$\mathrm{Bi}(x) = \frac{e^z u(t)}{x^{1/4}},$$

where $z = \frac{2}{3}\sqrt{x^3}$ and u is an expansion in $t = 2\left(\frac{18}{z}\right) - 1$.

For $|x| <$ ***machine precision***, the result is set directly to $\mathrm{Bi}(0)$. This both saves time and avoids possible intermediate underflows.

For large negative arguments, it becomes impossible to calculate the phase of the oscillating function with any accuracy so the function must fail. This occurs if $x < -\left(\frac{3}{2\epsilon}\right)^{2/3}$, where ϵ is the ***machine precision***.

For large positive arguments, there is a danger of causing overflow since Bi grows in an essentially exponential manner, so the function must fail.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x – double scalar**

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

x is too large and positive. On soft failure, the function returns zero.

ifail $= 2$

x is too large and negative. On soft failure, the function returns zero.

7 Accuracy

For negative arguments the function is oscillatory and hence absolute error is the appropriate measure. In the positive region the function is essentially exponential-like and here relative error is appropriate. The absolute error, E, and the relative error, ϵ, are related in principle to the relative error in the argument, δ, by

$$E \simeq \left|x\mathrm{Bi}'(x)\right|\delta, \epsilon \simeq \left|\frac{x\mathrm{Bi}'(x)}{\mathrm{Bi}(x)}\right|\delta.$$

In practice, approximate equality is the best that can be expected. When δ, ϵ or E is of the order of the ***machine precision***, the errors in the result will be somewhat larger.

For small x, errors are strongly damped and hence will be bounded essentially by the ***machine precision***.

For moderate to large negative x, the error behaviour is clearly oscillatory but the amplitude of the error grows like amplitude $\left(\frac{E}{\delta}\right) \sim \frac{|x|^{5/4}}{\sqrt{\pi}}$.

However the phase error will be growing roughly as $\frac{2}{3}\sqrt{|x|^3}$ and hence all accuracy will be lost for large negative arguments. This is due to the impossibility of calculating sin and cos to any accuracy if $\frac{2}{3}\sqrt{|x|^3} > \frac{1}{\delta}$.

For large positive arguments, the relative error amplification is considerable:

$$\frac{\epsilon}{\delta} \sim \sqrt{x^3}.$$

This means a loss of roughly two decimal places accuracy for arguments in the region of 20. However very large arguments are not possible due to the danger of causing overflow and errors are therefore limited in practice.

8 Further Comments

None.

9 Example

```
x = -10;
[result, ifail] = s17ah(x)
```

```
result =
   -0.3147
ifail =
           0
```